Decision, Uncertainty and Cooperation:

A Behavioral Interpretation Based On Quantum Strategy

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Abstract

This paper presents a new attempt to explore people's cooperative behavior under the natural uncertainty in decision making process. The most recent development of quantum cognition creatively enriched the exploration to the endogenous uncertainty in human behavior by expanding the strategic space. In this paper we extend the quantum decision-making model to a two-player Prisoner's dilemma game by bringing in the evolutional decision operators with quantum phases. These quantum phases, following uniform random distribution when the decisions are confined to individual decision-making process, will change according to the different levels of implied cooperative inclination, and lead to possible new Nash Equilibriums that emerge from a "hyper" decision space.

We also bring in the Dissimilarity Index from complex network literature, to capture and measure the cooperative inclination between the players. Conditions of Quantum Nash Equilibrium are derived out from dynamic quantum equations under the Heisenberg Picture.

Keywords: quantum strategy; cooperative inclination; dissimilarity index

1. Introduction

Cooperation and competition is one of the oldest pairs of paradoxes in market oriented economy. The origin and evolution of cooperation now becomes a cross-discipline research interest. With the ambition providing broader psychological foundation to decision making process, some updated findings in human cognition study, (Aerts,1995, 2009a, 2009b, 2011; Busemeyer, 2006a, 2006b, 2007,2011; Khrennikov, 2003, 2009, 2010; Pothos and Busemeyer, 2009; and Franco2009; etc.;) presented a novel framework with quantum strategies.

"Sure Thing Principle " (Savage 1954) predicts that given two independent observations, "One chooses action A over action B when she knows event E happens "and "She still choose A when she knows E does not happen", it can be logically derived out that the decision maker will stick to A regardless of knowing E happens or not. However, one of the famous violations to sure thing principles, "disjunction effect", (Tversky and Shafir,1992; Shafir and Tversky,1992) broke this prediction in experiment. Shafir and

Tversky conducted an experiment in which participants were asked to play a gamble of "50% chance to win 200 dollars or lose 100 dollars". The catch of the game is that the participants were told before start that they would have an opportunity to play the same gamble twice. The first round was obligatory, but they were allowed to decide whether play the second round. Research conductors studied the player's decisions on the second round game after their first ones played under three conditions: won, lost and not known the result. A majority of players chose to gamble the second round knowing that they won the first game (69%), so they did if knowing they lost the first game (59%); but they switched to choosing not to gamble the second round when they did not know the outcome of the first round (36%).

Another famous case for disjunction e□ect is Prisoners Dilemma Game (Fox and Tversky, 1995; Croson, 1999), experiment shows the ratio that players "irrationally" deviate from the optimal strategy (Defect) to the inferior strategy (Cooperation) will significantly rise, when the player does not know the move of the others for sure.

The recent development in quantum strategy theory provoked a more daring solution by taking the violation to sure thing principle as a special type of "decision interference", inspired by the well-known double-slit experiment of electron in quantum physics. In the above example, decisional strategies "Taking action A when (knowing) Event E happens " and "Taking action A (Knowing) Event E does not happen" are two vectors $|A_1\rangle$ and $|A_2\rangle$ in Hilbert space. Therefore the vector sum $|A_1\rangle + |A_2\rangle$ represents "Taking action A regardless knowing event E happens or not happen". $|A_1\rangle$ and $|A_2\rangle$ are complex numbers so we get

$$|A_1\rangle = |A_1|e^{i\theta_{A_1}}, |A_2\rangle = |A_2|e^{i\theta_{A_2}} \quad (1)$$

The probability of taking action A is the square of the modulus.

$$P(A)= \left| |A_1\rangle + |A_2\rangle \right|^2 = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\varphi_{A_2} - \varphi_{A_1}) \quad (2)$$

It is obviously that the probability of taking action A depends on the difference of phase angles φA.

$$\varphi_A = \varphi_{A_2} - \varphi_{A_1} \quad (3)$$

P(A) can be smaller than the sum of absolute values of $|A_1\rangle$ and $|A_2\rangle$ when φA takes some certain value. So the probability of taking action A not knowing exactly if E happens can be smaller than the simple addition of the probabilities of taking two independent strategies, which explains the violation to Sure Thing Principle.

Respectively, Aerts(2009a), Busemeyer(2006a), Khrennikov(2009), Franco(2009) are able to explain disjunction e□ect; Aerts(2009b), Khrennikov(2009) explains Allais paradox(Allais, 1953); Aerts(2011) explains Ellsberg paradox(Ellsberg,1961; Halevy,2007).

Quantum strategy is not imaginary fantasy made up by math trick. It is the reflection of the fact that human decision-making process is inherently uncertain and probabilistic. Vectors in Hilbert space are used to capture the intrinsic uncertainty, specifically, by identifying a proper quantum state vector for a particular decisional strategy. Busemeyer(2006a, 2006b, 2007),Pothos and Busemeyer(2009) compare quantum strategy method with Markov stochastic decision model (Regenwetter Falmagne and Grofman (1999); Ratcli and Smith(2004)). They found in the stochastic behavior models, psychological state of decision evolutes along only “single path” in all time, given the “one state” and “single path” is stochastic. However, for quantum strategy approach, psychological state of decision is captured by superposed state, while the evolution can pass along multiple paths simultaneously due to the fundamental characteristic of quantum. (Busemeyer (2006a)) So quantum strategy approach provides a superior and comparative vision for decision-making process despite the odd looks of mathematical form.

So far we briefly reviewed recent works in quantum strategy approach. It should be noticed that all these cited works are conditional on independent individual decision-making. The purpose of this paper is to generalize the quantum strategy approach and extend it to multiple players game.

2. Quantum Strategy and Strategic Operators

In the discussion in section 1, an individual quantum strategy takes form of

$$x|D\rangle + y|C\rangle \qquad (4)$$

$$|x|^2 + |y|^2 = 1 \qquad (5)$$

when x and y are complex coefficients.

Player k

Player j

	C	D
C	(r,r)	(s,t)
D	(t,s)	(p,p)

Table 1: Prisoner's Dilemma

We extend it to a two-players prisoner's dilemma game as in Table 1, which is, actually, a correlated strategic decision process of two persons. Table 1 is a generalized form of two players Prisoners Dilemma game. (payoffs will satisfy $2r > t + s > 2p$).We compose strategic operators (orthonormal vectors) to define strategy C (cooperate) and D (Defect) in Hilbert space as (6) and (7).

$$C=\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (6)$$

and

$$D=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (7)$$

Any mixed strategy in classical world could be written as a pair of ordinal real numbers shown in (8).

$$(p,1-p),0 \le p \le 1 \qquad (8)$$

p and $1-p$ are the probabilities of performing pure strategy C and D. For a quantum strategy, p is a complex number contains phase angles which extend (p, 1-p) to

$$(\cos^2\theta,\sin^2\theta),0 \le \theta \le \pi/2,0 \le \varphi \le \pi \qquad (9)$$

For simplification we study the square roots of (9).

$$(\cos\theta,-\sin\theta e^{i\varphi}),0 \le \theta \le \pi/2,0 \le \varphi \le \pi \qquad (10)$$

The real probability responds to the square of modulus, which is also used by Busemeyer, 2006a, 2007; Khrennikov 2009; Pothos, Busemeyer, & Franco 2009.

Now we compose the complete form of strategic operator:

$$\hat{S} = \begin{pmatrix} \cos\theta & e^{i\varphi}\sin\theta \\ -e^{i\varphi}\sin\theta & \cos\theta \end{pmatrix} \qquad (11)$$

Through this work we stick to the Heisenberg Picture in quantum dynamics, so the state vectors are fixed, not vary with time (while operators do). So it is convenient for us to lock the initial state vector as (6), and focus on developments of (11).

Property 2.1: Matrix in (11) is a unitary matrix (all elements are complex numbers; the modus of determinants equals 1).1

Property 2.2: (Any individual player's) quantum strategy is defined with parameter θ and φ.

Property 2.3: If ϕ = 0, quantum strategy degenerates to mixture strategy under classical condition.

Property 2.4: Operator matrix (11) is simultaneously a Hermite Matrix, whose eigenvalues are real numbers. This is a primary requirement of quantum computation operators.

As particular cases, pure strategy C (cooperate) is able to be expressed as:

$$\hat{C} \equiv \hat{S}(0,0), \hat{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (12)$$

While D (defect)

$$\hat{D} \equiv \hat{S}(\pi,0), \hat{D} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad (13)$$

When the quantum strategy is extended to two players' game, it is actually a correlated

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[1] In our model, any quantum strategy should be expressed as a unitary matrix, to ensure the total probability as 1.

interactive decision-making process. The total strategic space of the two players possesses four orthogonal base vectors: |DD⟩, |CD⟩, |DC⟩|CC⟩. To be consistent with previous discussions, we let |DD⟩ as column vector (14):

$$|DD\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad (14)$$

Transformation in (14) also applies to three other base vectors. Quantum strategy can be rewritten as the product of the base vector premutiplied with a 4 × 4 unitary matrix.

Each player has her own strategic operator $\hat{S}_j(\theta_j, \varphi_j)$和$\hat{S}_k(\theta_k, \varphi_k)$ given every strategic operator is a 2 × 2 unitary Hermite matrix working on a specific Hilbert strategic space of her own. The joint effect becomes the tensor product of the two individual strategic operators:

$$\hat{S} = \hat{S}_j \otimes \hat{S}_k \qquad (15)$$

Under Heisenberg Picture (Dirac, 1931), (14) is fixed as initial state vector being constant to time variation. So it is not hard to verify, that if we premultiply (15) to (14), and let $\phi j = 0, \phi k = 0$, modulus squares of the four components of the vector will respectively be:

$$\cos^2\theta_j \cos^2\theta_k,\ \cos^2\theta_j \sin^2\theta_k\ ,\ \sin^2\theta_j \cos^2\theta_k\ ,\ \sin^2\theta_j \sin^2\theta_k$$

Suppose we let $\cos^2\theta_j = p, \cos^2\theta_k = q$, the quantum strategic operator would degenerate to the probability distribution of mixture strategy in classic status: p q, p(1 − q), (1 − p)q, (1 − p)(1 − q). Otherwise, ϕ1, ϕ2 nonzero, strategic phase angle difference will exist, which means the quantum strategy should hold.

The angle phase difference between two players will capture how correlative factors, like cooperation inclination, work on players' strategy selection, and ultimately the equilibrium of the game. Take ϕ1, ϕ2 as non-independent or interacted factors, in the players total strategic space, phase di□erence is interference of two players' quantum strategic wave functions. One consequence of interference is that strategic operator will evolve.

Equation (16) gives out the form how evolution operator $\hat{U}$ takes effect on the total strategic operator $\hat{S}$, shown as

$$\hat{S}_{evo} = \hat{U}^{+} \hat{S} \hat{U} \quad (16)$$

$\hat{U}^{+}$ is a conjugate transposed matrix to $\hat{U}$.(Under **Schrodinger Picture,** it is equivalent to two players' total wave function being affected by interaction potential).The time evolution operator is a Hermite Matrix, in which the elements will carry implications related to the correlation and cooperative inclination variables.

3 Cooperative Inclination and Dissimilarity Index

Sally (2002) discussed the existence of compassion and sympathy between players by introducing an endogenous function. In a Prisoners Dilemma game in table 1, to capture the general motional connections between the players in a Hilbert strategy space, we compose a function of “cooperative inclination” .

A main challenge is how the players’ cooperative inclination twisted by physical and psychological distance. We employ the approach of Dissimilarity Index (Zhou, 2003a, 2003b) from Complex Network literature (Jackson, 2008; Goyal,2007). Another reason for borrowing dissimilarity index, is that it allow the distance between two nodes to be asymmetric.

$$ {}_j\lambda_k = 1 - \frac{\omega\varphi_{jk} + (1-\omega)\psi_{jk}}{\delta} \qquad (17) $$

In equation (17), ${}_j\lambda_k$ measures for the cooperative will player j holds for player k. The cooperative degrees are jointly determined by the physical distance and psychological distance they feel for each other. φ_{jk} and ψ_{jk} physical and psychological distance between player j and player k respectively, δ represents the maximal distance between two players, ω is the weight attached to each type of distances .

Granted ${}_j\lambda_k \in [0,1]$, 0 for no cooperative inclination, while 1 for the largest inclination, equation (17) shows that the smaller of both the two distances, the stronger the cooperation inclination.

For any particular game, real effect of cooperation is defined by the cooperative inclination held and perceived reciprocally. Cooperative function $\Lambda({}_j\lambda_k, {}_k\lambda_j)$ characterizes this understanding, with the fundamental properties of:

Property 3.1: $\Lambda({}_j\lambda_k, {}_k\lambda_j) \le 1$ (18)

Property 3, 2A: $\dfrac{\partial\Lambda({}_j\lambda_k, {}_k\lambda_j)}{\partial\, {}_j\lambda_k} > 0$ (19)

$$\frac{\partial\Lambda({}_j\lambda_k, {}_k\lambda_j)}{\partial\, {}_k\lambda_j} > 0 \quad (20)$$

i.e, To increase (decrease) any player's cooperation degree will increase (decrease) the value of cooperation function.

Property 3.2B: $\Lambda({}_j\lambda_k, {}_k\lambda_j) \le \max({}_j\lambda_k, {}_k\lambda_j)$ (21)

Property 3.2C: $\Lambda(\lambda, \lambda) = \lambda$ (22)

Property 3.3: $\Lambda(0, {}_k\lambda_j) = 0$ (23)

i.e, if any player's cooperation degree is 0, the value of cooperation function will be non-positive number (for simplicity, we let it be 0).

Property 3.4: $\Lambda({}_j\lambda_k, {}_k\lambda_j)$ is continuous for all ${}_j\lambda_k$ and ${}_k\lambda_j$.

A fuction form satisfying 3.1-3.4 is the geometric mean of ${}_j\lambda_k$ and ${}_k\lambda_j$.,

$$\Lambda({}_j\lambda_k, {}_k\lambda_j) = \sqrt{({}_j\lambda_k)({}_k\lambda_j)} \quad (24)$$

(For reading convenience, we simplify $\Lambda({}_j\lambda_k, {}_k\lambda_j)$ with $\Lambda(j,k)$ in after discussions.)

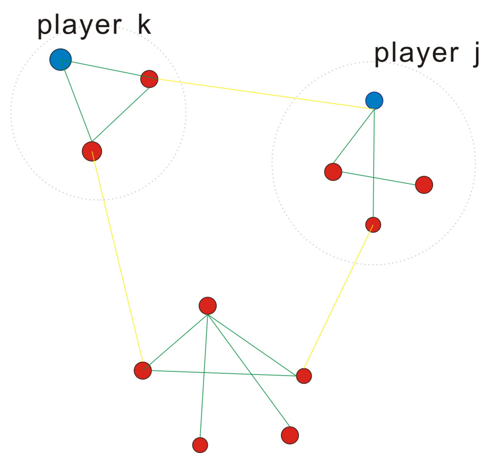

Figure 1: The structure of network communities

Dissimilarity index is rooted from the concept of Euclidean distance under social network study context. It is a good measure of the "distance" between two nodes caused by the different affiliation relationship. (Wasserman and Faust,1994). The complete network is made of various groups and clusters with denser conjunctions within the internal vertex, which share closer value and recognized standards, more likely to react consistently to certain stimulations from externality. Comparatively, the bindings between those groups or clusters are sparse. Figure 1 shows the structure of connecting framework within and between groups.

(25) defines the Euclidean distance between two nodes k and j in a complex network. In an adjacency matrix A, A_{jk} values 1 if j and k share at least one edge connection, otherwise, 0. Column (line) j in the adjacency matrix shows the connectivity between j with all the others:

$$x_{jk} = \sqrt{\sum_{l \neq j,k} (A_{jl} - A_{kj})^2} \quad (25)$$

x_{jk} in (25) is a measurement of similarity, or the degree that two vertices in the whole network are equivalent in structural function.

Another widely used measure is Pearson Correlation, displays the statistical correlativity between two columns in the adjacent matrix.

$$x_{jk} = \frac{\sum_l (A_{jl} - \mu_j)(A_{kl} - \mu_k)}{n\sigma_j \sigma_k} \quad (26)$$

given $$\mu_j = \frac{\sum_k A_{jk}}{n}, \ \sigma_j = \sum_k (A_{jk} - \mu_k)^2$$

Zhou (2003a, 2003b) improved dissimilarity index by applying random walk method to rebuild the intra-nodes distance. Now "Distance" between j and k is defined as the number of random walk steps which are needed for a Brownian particle to move from j to k. Assume a Brownian particle jumps to an adjacent node position, the probability it reaches a given adjacent k is:

$$P_{jk} = \frac{A_{jk}}{\sum_{l=1}^{N} A_{jl}} \quad (27)$$

After jumping a large number of steps ($\rangle\rangle n$), the probability for this particle to reach k is:

$$\rho_k = \frac{\sum_{l} A_{kl}}{\sum_{m,n} A_{mn}} \quad (28)$$

From (27) and (28), the final number of steps can be solved out through a series of linear algebra calculation:

$$d_{j,k} = \sum_{l=1}^{N} \left(\frac{1}{I - B(k)}\right)_{jk} \quad (29)$$

I is $N \times N$ unit matrix, $B(k)$ is matrix transformed from P in (27) replacing column k with column vector 0. Solving linear algebra $[I - B(k)]\{d_{1,k},\ldots,d_{N,k}\}^T = \{1,\ldots,1\}^T$, all $d_{j,k}$ can be worked out. Then we are able to compose Dissimilar Index ${}_j\Im_k$:

$$ {}_j\Im_k = \frac{\sqrt{\sum_{l \neq j,k}^{N} (d_{jl} - d_{kj})^2}}{(N-2)} \quad (30) $$

(30) is the variance between two number groups $d_{j,k}$ and $d_{j,l}$.Consequently, given dissimilarity index ${}_j\Im_k$ smaller , the overall picture of the network from the view of nodes k and j are more similar. Dissimilarity index takes value in the close interval [0, 1].

Then we let

$$ {}_j\lambda_k = 1 - {}_j\Im_k \qquad (31) $$

Now obtain the expression format of cooperative inclination $\Lambda(j,k)$ in the precious discussion.

4 Two Players Model with Cooperation Inclination

As discussed in section 3, $\Lambda(j,k)$ is the function of intended cooperative inclination**,** then elements of the evolution matrix $\hat{U}$ can be written as function of $\Lambda(j,k)$ i.e, $\hat{S}_{evo} = \hat{S}_{\Lambda(j,k)}$, given $\hat{U}$ satisfying the conditions of:

Property 4.1. : Unitness

$\hat{U}$ degenerates into unitary matrix if $\Lambda(j,k) = 0$.

Property 4. 2. : Smoothness

All elements of $\hat{U}$ are continuous differentiable function of $\Lambda(j,k) = 0$.

Corollary:

If $\Lambda(j,k) = 0$ is a infinitesimal , taking the form of :

$$\hat{U}_{\Lambda(j,k)} = I + \Lambda(j,k)X + 1/2\Lambda(j,k)^2 X^2 \qquad (32)$$

then $$X \equiv \frac{\partial \hat{U}}{\partial \Lambda(j,k)}_{\Lambda(j,k)=0} \qquad (33)$$

$$X^2 \equiv \frac{\partial^2 \hat{U}}{\partial^2 \Lambda(j,k)}_{\Lambda(j,k)=0} \qquad (34)$$

Proof: $\hat{U}$ is able to follow Taylor Expansion under matrix form because of smoothness. Take the two items from the expanded expression to obtain (32).

Property 4.3. : Completeness

Total effect of two time evolution processes with $\hat{U}_1$ and $\hat{U}_2$ on strategic matrix $\hat{S}$ can be reform into one time evolution process $\hat{U}_3$:

Corollary: $Tr(X) = 0$, which means matrix X is traceless.

Proof: Assume $\Lambda(j,k)$ is an infinitesimal, we have:

$$\delta\Lambda(j,k) \equiv \frac{\Lambda(j,k)}{N}, \lim N = \infty \quad (35)$$

Repeat the evolution operation $\hat{U}$ for n times, from the completeness condition we can have:

$$\hat{U}_{\Lambda(j,k)} = \lim_{N\to\infty}[\hat{S}_{\delta\Lambda(j,k)}]^N = \exp(\Lambda(j,k)X) \quad (36)$$

Take the determinant $\hat{U}$, because $\det \hat{U} = 1$, easy to find that $Tr(X) = 0$.

Property 4.4.: Classicality Perseverance

The four classical strategies DD, DC, CD, CC are inclusive in $\hat{S}_{evo} = \hat{S}_{\Lambda(j,k)}$

$$\hat{U}^{+}(D \otimes D)\hat{U} = D \otimes D \quad (37)$$

(38) is verified to be the only satisfying matrix

$$\begin{pmatrix} \sqrt{1-[\Lambda(j,k)/2]^2} & i\Lambda(j,k)/2 & 0 & 0 \\ i\Lambda(j,k)/2 & \sqrt{1-[\Lambda(j,k)/2]^2} & 0 & 0 \\ 0 & 0 & \sqrt{1-[\Lambda(j,k)/2]^2} & i\Lambda(j,k)/2 \\ 0 & 0 & i\Lambda(j,k)/2 & \sqrt{1-[\Lambda(j,k)/2]^2} \end{pmatrix} \quad (38)$$

5. Calculation of Expected Payoff Value and Deduction of Nash Equilibrium

Premultiply total strategic operators $\hat{S}_{tol} = \hat{S}_{evo}$ on initial state vector $|DD\rangle$:

$$|\phi_f\rangle = \hat{U}|DD\rangle \quad (39)$$

$|\phi_f\rangle$ has four components：

$$|\phi_f\rangle = \begin{pmatrix} \phi_{CD} \\ \phi_{DC} \\ \phi_{DD} \\ \phi_{CC} \end{pmatrix} \quad （40）$$

Each component is one projection of $|\phi_f\rangle$ on each of the 4 orthogonal basic vectors. The square of complex modulus of each vector component composes the expected coeffcient of corresponding payoff value:

$$\$_A = rP_{CC} + pP_{DD} + tP_{DC} + sP_{CD} \quad (41)$$

$$P_{CC} = \left|\phi_{CC}\right|^2 \text{, etc.}$$

It should be emphasize here that only the squares of complex modulus of 4 components in $\left|\phi_f\right\rangle$ correspond to probability in reality. So the quantum strategy $\hat{S}_{tol}$ and the initial state vectors are technically only the hyper-intermediate processes that do not directly apply to the players' move in reality.

To give a calculation example, for the Prisoners Dilemma game in table 1, given $\Lambda(j,k) = 1$, The quantum expected payoff value of player j (Alice) now is:

$$P_j(\theta_j,\phi_j,\theta_k,\phi_k) = r\left|\cos(\phi_j + \phi_k)\cos(\theta_j/2)\cos(\theta_k/2)\right|^2 \qquad (42)$$

$$+ p\left|\sin(\phi_j)\cos(\theta_j/2)\sin(\theta_k/2) - \cos(\phi_k)\cos(\theta_k/2)\sin(\theta_j/2)\right|^2$$

$$+ t\left|\sin(\phi_k)\cos(\theta_j/2)\sin(\theta_k/2) - \cos(\phi_j)\cos(\theta_k/2)\sin(\theta_j/2)\right|^2$$

$$+ s\left|\sin(\phi_j + \phi_k)\cos(\theta_j/2)\sin(\theta_k/2) + \sin(\theta_j/2)\sin(\theta_k/2)\right|^2$$

Now we look into the Nash equilibrium on expected quantum pay off values. Define quantum Q strategy as (43) (Eisert, Wilkens, and Lewenstein (1999)):

$$Q \sim U(0,\pi/2) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \qquad (43)$$

If one player chooses strategy D, for the other player, choosing strategy Q will offer higher payoff value than D. Naturally, suppose two players start with strategy set (U1(θ1,ϕ1), U2(θ2,ϕ2)), player 1 will change strategy from U1 to Q for improving gains unless U1 = Q .The same is for player2. On the contrary, suppose two players start with strategies (Q,Q), no one will deviate from strategy Q alone. So (Q,Q) is a Nash equilibrium. But strategy Q is NOT performable like a classic strategy. It is pure quantum strategy. Substitute $(\hat{Q}_A \otimes \hat{Q}_B)$ into (42), easy to verify only the coeffcient before r is nonzero, which means the projection of (Q, Q) on coordinate unit vectors is (C, C).

To simplify the calculation, we define:

$$\gamma = \arcsin \Lambda(i, j) \qquad (44)$$

Then the quantum expected payoff of player j (Alice) is:

$$\begin{aligned}
P_j(\theta_j, \phi_j, \theta_k, \phi_k) = r\big|&(\cos(\phi_j + \phi_k)\cos(\theta_j/2)\cos(\theta_k/2))^2 \\
&+ \cos(\theta_j/2)(\cos(\theta_k/2))^2(\cos(\gamma))^2(\sin(\phi_j + \phi_k))^2\big| \\
&+ p\big|2\sin(\phi_j)\sin(\theta_j/2)\cos(\theta_k/2)\cos(\gamma/2)\sin(\gamma/2) \\
&+ \cos(\phi_j)\cos(\theta_j/2)(\cos(\theta_k/2))^2 \\
&(\sin(\phi_j)\cos(\theta_j/2)\sin(\theta_k/2)^2)\big| \\
&t\big|(2\sin(\phi_j)\cos(\theta_j/2)\sin(\theta_k/2)\cos(\gamma/2)\sin(\gamma/2) \\
&- \cos(\phi_k)\cos(\theta_k/2))^2
\end{aligned}$$

$$+\sin(\phi_k)\sin(\theta_j/2)\cos(\theta_k/2)\cos(\gamma)^2\Big|$$

$$+s\Big|2\sin(\phi_j+\phi_k)\cos(\theta_j/2)\cos(\theta_k/2)\cos(\gamma/2)\sin(\gamma/2)$$

$$+(\sin(\theta_j/2)\sin(\theta_k/2))^2\Big| \qquad (45)$$

The quantum expected payoff value of player k (Bob) is calculated in same method. In general, there exists a certain critical value for $\Lambda(j,k)$ to make (Q,Q) a new quantum Nash equilibrium. The observable outcome in reality is the player's shift from strategy (D, D) to (C, C).

For example, let r=3, t=5, s=0,p=2 (Table 1)

$$\$_A(\hat{U}(\theta,\phi),\hat{D}) = \sin^2(\theta/2) + 5\cos^2(\theta/2)\sin^2\phi\sin^2\gamma \qquad (46)$$

however,

$$\$_A(\hat{U}(\theta,\phi),\hat{Q}) = 4-\cos\theta + (-3+2\cos\theta-\cos^2(\theta/2)\cos 2\phi)\sin^2\gamma \qquad (47)$$

Solve out the critical value: $\gamma_{th} = \arcsin\sqrt{2/5} \approx 0.685$, which is corresponding to cooperation inclination function value $\Lambda(j,k) = 0.4$ Emergence of the new Nash Equilibrium is a consequence of quantum conditions, specificly, conditional on phase factor φ. If $\varphi = 0$, there will be no new Nash equilibrium.

6. Summary and conclusions

In this work we studied how cooperation is possible with the intrinsic uncertainty of human behavior, especially in competitions and confrontation like Prisoners dilemma type. The conditions for cooperation are more complex in a quantum game because they depend on both the potential intra- connections and the existence of a high dimensional strategic space. If people actually had been making decisions in a bigger space th-an the one we use to take for granted, quantum strategies then would emerge from the shadow of irrationality. Quantum Nash Equilibrium is possible, even though the actions taken in “real world” are only their projections in a lower dimensional world.

Behavioral economics provides an open framework to retain, test and incorporate elements and factors like emotion, religions ideology, culture and psychological e□ects. If we realize the similar “actions” observed could embraced how di□erent motivations and resolutions, if we can accept the endogenous complexity in mind and behavior patterns, it might be more comfortable to accept the sustainable “irrationalities”. They could be no more than the overlapped shadows of “rationalities” from simply bigger decision space.

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