# Misperception of Risk and Incentives by Experienced Agents*

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May 10, 2014

**Abstract**

We study an observable risk-return tradeoff for which risk "preferences" are normatively prescribed by the desire to win the game. The choice is whether to shoot a 2-pointer or a 3-pointer in professional (NBA) basketball. When trailing, teams should get more risk-loving as they fall further behind, matching prospect theory preferences. When leading, risk aversion should increase with the lead, running counter to typical risk preferences. We find strong evidence that players trade off risk and return correctly only in the trailing domain. In the leading domain, they incorrectly exhibit decreasing absolute risk aversion with the magnitude of their lead. Players thus exhibit preferences similar those widely found in the lab, but misapplied in this setting.

*Jim Andreoni, David Eil, Uri Gneezy, Ben Handel and Grant Wang gave us detailed comments and feedback. A subset of these results were presented and distributed at the MIT Sloan Sports Analytics Conference under the title "The Price of Risk in the NBA."

# 1 Introduction

Everyone has risk attitudes that they use to go about their daily life. In many settings, these day-to-day preferences run counter to one's or one's employer's long-run best interests. For example, Benartzi and Thaler (1995) argue that excess risk aversion in long-run personal investments is explained by excessive focus on year-over-year returns. These investors, however, are typically inexperienced and lack risk management expertise. Such caveats are not relevant to a firm manager who brings personal risk preferences to bear on firm choices, creating a well-known agency problem (Fama, 1980). This is more than a theoretical concern because in controlled experiments firm managers tend to be risk-loving over losses and risk averse over gains (Laughhunn et al., 1980; Fiegenbaum and Thomas, 1988). Since it is generally impossible to observe the (ex-ante) gambles managers face it is unclear how pervasive this bias is in practice. A critical question is thus: to what extent are the risk preferences employed by experienced agents in the field biased by their day-to-day disposition towards risk?

We tackle this question by studying an observable risk-return tradeoff made by highly experienced professionals: National Basketball Association (NBA) players selecting between 2-point and 3-point shot attempts. Four important features of their strategic environment make it well suited for this purpose. First, players have ample experience with the decision. Second, the high stakes nature of professional sports mean millions of dollars are often on the line. Third, and perhaps most importantly, we need not conjecture as to what constitutes a choice mistake since risk "preferences" are entirely prescribed by the desire to win. Fourth, the risk preferences players should adopt differ starkly from those widely found through experimental elicitation in the laboratory and field.

We define a shot's real value as the increase in win probability for the shooting team if the shot is made. The real value of a 3-point shot relative to a 2-point shot depends primarily on the score margin and time-remaining in the game. Early in the game, when many scoring opportunities remain, teams should maximize expected points—a 3-pointer is simply worth one-and-a-half 2-pointers. While 3-pointers have approximately double the per-shot variance, at this stage the team is best served by risk neutrality—per-shot variance has a lower order contribution to win probability than expected point value. When there are relatively few remaining scoring opportunities, approximately the last one-third of the game, these asymptotics begin to lose bite. Now the trailing team should place an increasingly positive value on per-shot variance as time remaining decreases and as they fall further behind in order to induce larger swings in the final point total. Conversely, the leading team should place an increasingly negative value on per-shot variance as their lead increases and time remaining decreases.

To see how the requirements of optimal play map to typical risk preferences, note that as a trailing team falls further behind it should get more risk-loving, just as in prospect theory.[1] When the game is tied, even with very little time remaining, a team should be effectively risk neutral and

[1]Rather than cite one of many review articles, we note that Kahneman and Tversky (1979) has more than 25,000 citations according to Google Scholar.

should become increasingly risk-averse as it moves into the lead. Both of these properties violate decreasing absolute risk aversion, a robust feature of individual preferences over gambles (Friend and Blume, 1975; Robson, 2002). Thus, the risk attitudes players should exhibit match (typical) day-to-day preferences in the trailing domain but run counter to them in the leading domain.

In order to derive testable predictions of adherence to these risk dynamics, we model the strategic interaction as a two-by-two attacker/defender game. The offense chooses shot mixture and the defense simultaneously allocates scarce "attention" to guard against each shot type. With very minimal assumptions, we establish the intuitive condition that 3-point efficiency (points scored per shot) must be inversely correlated with a team's preference for risk—when a team should become more risk loving (averse), optimal shot selection implies that 3-point success rate *must* go down (up). We test this optimality condition using detailed play-by-play data from four complete NBA seasons.[2]

Our key finding is that teams correctly trade off risk and return *only in the trailing domain* (all results stated are significant at the 0.001 level). When the losing team should become increasingly risk-loving, players shoot more 3-pointers and at decreasing expected nominal point value compared to 2-pointers taken in such situations. This pattern is consistent with optimal play. In contrast, when the leading team should become more risk averse, as the should when they move further ahead, they take significantly *more* 3-pointers and these shots return significantly *fewer* points in expectation. The opportunity cost, or price, of a 3-pointer is the expected real value of a forgone 2-pointer. Restating our results, the leading team pays an increasingly higher price for a gamble—a 3-pointer—that is becoming less valuable in real terms. In the leading domain players invert the price of risk, in the trailing domain they respect it. This finding is quite striking: one could have more accurately predicted behavior of these experienced agents in a high stakes environment by reading Kahneman and Tversky (1979) than by accurately modeling the risk-return tradeoff.

There is a deep literature on the role of expertise and incentives in strategic decision making (see Dellavigna (2009) for a review). Papers typically start with a behavioral pattern observed in the laboratory and then study it in the field where experience and stakes tend to be higher. For example, trading experience can eliminate the endowment effect (List, 2003), but does not entirely remove the disposition effect (Feng and Seasholes, 2005). It is difficult to apply this framework to risk preferences because the line between choice mistakes and a faithful representation of true preferences is generally blurry. Instead normative evaluation comes from an agent's felicity in maximizing the preference function (Choi et al., 2007) or the consistency of choice across similar domains, such as purchasing auto and home insurance (Friedman and Savage, 1948; March and Shapira, 1987; Barseghyan et al., 2011; Dohmen et al., 2011).[3] Our setting possesses two critical

[2]We only use game states in which each team has a predicted chance of winning the game greater than 5%. Finally, we eliminate *fast break* possessions used with more than 14 seconds remaining on the shot clock.

[3]Related work on "choice bracketing" has asserted that often people use "narrow frames" to evaluate "broad frame" decisions, but again it can be difficult to draw the line between mistakes and preferences (Simonson and Winer, 1992; Benartzi and Thaler, 1995). Pope and Schweitzer (2011) conclude that professional golfers exhibit narrowly bracketed loss aversion (a day-to-day preference golfers would be better off suppressing). Our results also speak to the recent debate about how well expertise and preferences travel across contexts. Professional athletes appear

features that allow us to apply the standard framework: 1) the risk preferences agents should display are theoretically pinned down 2) the empirical variance and success rates of gambles are observable.[4] We find that our experienced agents behave rationally only when the prescribed risk preferences align with those found more broadly in prospect theory. When they do not, players express "everyday" risk attitudes despite the fact that this disposition runs counter to the team's interests—experience does not appear to be sufficient to eliminate bias in these agents.

## 2 Quantifying a Team's Objective Function

Our analysis is based on the minimal assumption that a team's goal is to win. One might be concerned that a lousy team might try to intentionally lose, or "tank," late in the season to improve their chance of receiving a high draft pick. Work on the topic suggests this is very rare (Price and Wolfers, 2010). Within a game, the incentive to win might fade when the game is out of reach, so we are careful to exclude situations in which one team has less than a 5% chance of winning. We also eliminate "fast-breaks" (shots taken very quickly after the other team has turned the ball over, shot clock $> 14$) and end of quarter shots, as these types of shots tend to have very different strategic considerations.

We use the term "win value" to refer to the impact a given action has on the probability a team wins the game and the term "nominal value" for the number of points scored. The three most important factors that determine win value at a given game state are the score margin, time remaining and possession of the ball. The increase in win value of adding 2 or 3 points to the team's current score are denoted $WV_2$ and $WV_3$ respectively. An intuitive estimation approach for these quantities is a non-parametric procedure, which takes a large number of games at a given game state $X$ and compares the probability of winning at $X$ to a nearby state $X'$. However, even with many seasons of data this procedure generates relatively noisy estimates of the slope and curvature of the win probability surface. We instead employ a parametric procedure that directly models the impact of team quality, pace of play, margin, time remaining and other relevant factors to predict win probability for a given game state. We describe it in detail the Appendix. Figure 1 shows that this method yields very similar win probability projections to those attained by nonparametric estimation. Given the far greater smoothness, we use the parametric estimates from here on out.

The relationship between the win value of 3-pointers and 2-pointers can be represented by a parameter that we call $\alpha$, defined as: $\alpha = \frac{WV_3}{WV_2}$. It measures how much a 3-pointer's win value diverges from that of one-and-a-half 2-pointers. When $\alpha > 1.5$, the win value of a 3-pointer exceeds it's nominal value. This occurs for the trailing team, especially late in the game, which can be seen in the convexity in the trailing domain of Figure 1. When $\alpha < 1.5$ the opposite is true, a 3-pointer is worth less than its nominal value—the team should be risk averse—as seen in the concavity of

capable of correctly mixing in 2x2 matching pennies style games when playing their sport (Walker and Wooders, 2001; Chiappori et al., 2002), but this ability does not appear to generalize to the lab (Levitt et al., 2010).

[4]Walls and Dyer (1996) look at ex-post revealed risk preferences in firm purchases of drilling rights. They discuss challenges associated with estimating the necessary probabilities and economic returns when studying firm decisions.

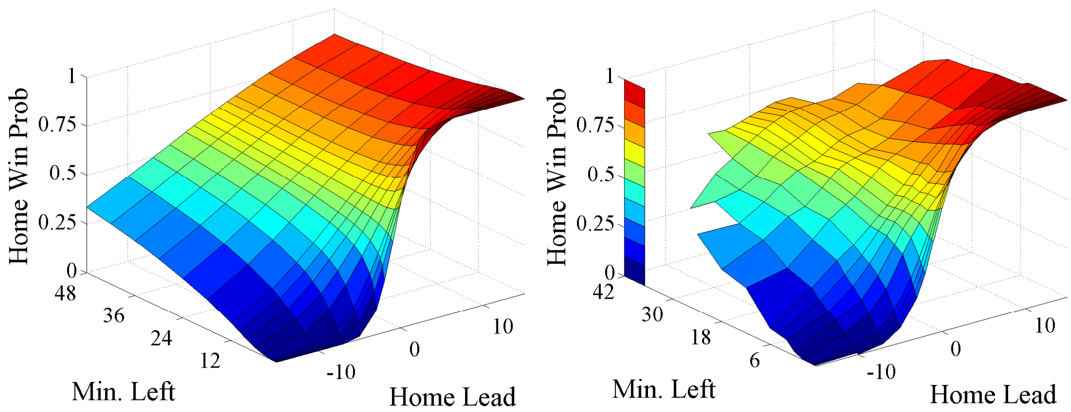


Figure 1: Parametric projects of win probability conditional on score margin and time remaining for the home team in even match-up; Panel 2: non-parametric estimates of the same function.

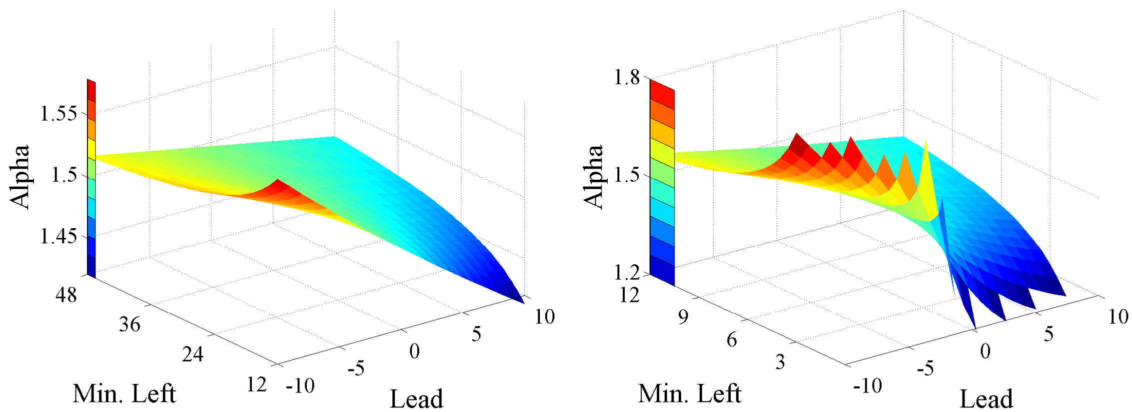


Figure 2: $\alpha$ as a function of game state: Quarters 1-3 (left) and Quarter 4 (right).

the win probability function in the leading domain.

Across all game states $\alpha$ is monotonically decreasing with a team's current lead, which is easy to see in Figure 2. Panel 1 shows the first three quarters for even strength teams on a neutral court. In the first half $\alpha$ is always close to 1.5—teams should be very close to risk neutral at all times. In the third quarter (minutes 12-24 remaining) we see more variation; $\alpha$ is between 1.4 and 1.6 provided the margin is less than 11 points. Panel 2 shows the fourth quarter (note the change in y-axis scale), which shows $\alpha$ varying widely. With fewer possessions remaining, the trailing (leading) team's preference for risk increases (decreases) dramatically.[5]

[5] $\alpha$ is a natural proxy for a team's preference for risk because it maps directly to the relative preferences over potential outcomes. Consider a case where $p_2 = 0.50$ and $p_3 = 0.33$. In this case, each shot has an expected nominal value of one point. The variance in the return of a 3-point attempt is $3^2 * .33 * .66 = 1.96$ and for the two-pointer $2^2 .5^2 = 1$. Suppose $\alpha = 1.7$. This means the expected utility (expected real value) of a 3-pointer is $0.33{*}1.7{*}WV_2{=}0.561{*}WV_2$ and 2-pointer is worth $0.5 * WV_2$; or in other words, the 3-pointer is worth 12% more in win value, despite having equal nominal value. If we model the team as having preference over mean and variance, we could map any $\alpha$ to a utility value of variance. However, we view $\alpha$ as a more directly interpretable parameter.

# 3 The Shot Type Allocation Game

We express the offense's optimization problem as a function of $\alpha$ and solve for the optimal mix of 2 and 3-pointers for each game situation. We will employ a concept in basketball analysis called the "usage curve" (Oliver, 2004; Skinner, 2011). The usage curve relates the frequency of a given shot type (2- or 3-pointer) to success rate and are naturally assumed to be downward sloping, implying that as a team shoots more 3-pointers the success rate on each successive 3-point attempt goes down and the success rate on 2-pointers goes up.[6] Let $\phi(u_3)$ denote the average probability of success for 3-pointers when the fraction of shots attempted as 3's is $u_3 \in [0, 1]$. $\psi(1 - u_3)$ gives the corresponding average probability of success on 2-point attempts.

We'll start by solving the model with no defensive adjustment. Since $WV_3 = \alpha WV_2$, we can write the team's maximization problem as:

$$\max_{u_3} u_3 * \phi(u_3) * \alpha WV_2 + (1 - u_3) * \psi(1 - u_3) * WV_2. \tag{1}$$

The first order condition can be rearranged to give:

$$\alpha(\phi(u_3) + \phi'(u_3) * u_3) = \psi(1 - u_3) + \psi'(1 - u_3)(1 - u_3). \tag{2}$$

The left side of equation (2) gives the marginal returns to shooting a 3-pointer. Shooting an extra 3-pointer returns the current average value ($\alpha * \phi[u_3]$), but it also impacts the average value of all the other 3-pointers taken ($u_3$) by a degree given by the slope of the usage curve ($\phi'[u_3]$). The right hand side gives the marginal returns to shooting a 2-pointer and can be understood with similar logic. In this model with no defensive adjustment, $u_3$ is increasing with $\alpha$. To see this note that if $\alpha$ increases then the left side goes up because the term $\phi(u_3) + \phi'(u_3) * u_3$ has to be positive, otherwise the marginal 3-pointer nets negative value. So the left side must increase, to counter-act this the right side must go up as well, which occurs only when $u_3$ increases.

Appendix Figure 1 gives a graphical representation of the maximization problem and the impact of an increase in $\alpha$. It is equivalent to a monopolist firm producing two goods with downward sloping demand and zero production costs from a unit of common input. An increase in $\alpha$ shifts up the "demand" for 3-pointers as a tax-rebate would. Just as the monopolist would not set the same price for the goods unless the demand curves were identical, optimal shot choice does not imply 2-pointers and 3-pointers offer the same average point value. The difference in average shot value is determined by the slope and intercepts of the usage curves. In practice 3-pointers tend to return more points-per-shot on average and are shot less often than 2-pointers, together implying a usage curve with a higher y-intercept and a steeper slope.

We now formally state our first proposition.

[6] A micro-founded explanation for this assumption comes from an optimal stopping model, as given in Goldman and Rao (2013). If we model the offense as getting shot opportunity arrivals over the course of a 24-second shot-clock, then to take more 3's, the team has to accept lower quality 3-point opportunities on the margin.

**Proposition 1** *In the model with no defensive adjustment, as $\alpha$ increases the fraction of 3-pointers attempted ($u_3$) goes up, the nominal value of attempted 3-pointers goes down, the nominal value of attempted 2-pointers goes up and the real value of attempted 3-pointers goes up.*

*Proof: See Appendix*

The model without defensive adjustment can be interpreted as representing a world in which defensive adjustments matter relatively little. Incorporating defensive adjustments is not difficult; the defense's objective is simply the opposite of the offense's (it wants to minimize equation (1)—an increase in the value of 3's increases the incentive to defend against them). We assume the defense has a unit of "defensive resources," which it can apply to defending 2's and 3's: $d_2 + d_3 = 1$. More defensive attention lowers the success rate of a shot type. We modify the usage curves to include defense $(\phi(u_3, d_3), \psi(u_2, d_2))$. Analysis of this model is straightforward, but algebraically involved, so we have placed it to the Appendix. We now state our second and third propositions:

**Proposition 2** *In the model with defensive adjustment, as $\alpha$ increases the average nominal value of attempted 3's falls and the average nominal value of attempted 2's rises.*

*Proof: See Appendix*

**Proposition 3** *In the model with defensive adjustment, as $\alpha$ increases the change in the usage rate of 3's is ambiguous. It depends on the slope of the 3-point usage curve, the impact of defense on the marginal shot values and the concavity of the usage curves with respect to defensive pressure.*

*Proof: See Appendix*

Proposition 2 states that the no-defense model's prediction that the average nominal value of of 3-pointers falls as $\alpha$ increases carries through. Proposition 3 states that the shot type frequency prediction is not robust to allowing a large class of defensive pressure adjustments. With defensive adjustment, the offense will shoot more 3's provided the defense cannot adjust pressure efficiently enough to discourage these additional attempts.

Our final extension of the model is to allow for a multiplicative function of $\alpha$ on each usage curve that accounts for a possible motivational impact of being behind in the game, as in Berger and Pope (2011).[7] It is easy to show that this term will cancel out of the first order conditions. However, we must amend Proposition 2 to be:

**Proposition 4** *If we allow for a motivational impact of trailing and defensive adjustment, then as $\alpha$ increases, the average nominal value of 3-pointers falls* ***relative*** *to the average nominal value of 2-pointers.*

[7] We could also model this as an additively separable term with the same impact on shot selection

When 3's become more valuable, maximization implies that the efficiency of 3's must fall relative to 2's. This is our most robust prediction, as it is true in the very general defensive adjustment setting and when allowing for an extra motivational impact of being behind. Respecting the returns to risk *requires* that Proposition 4 holds.

One concern is that we have modeled the team as greedily maximizing the probability winning the game at each possession. We are not allowing them to change the expected plan for shooting decisions on future possessions. This assumption limits the ability of our model to provide accurate (fully rational) point estimate predictions of optimal adjustment. For instance, if we could measure the *marginal* efficiency functions for 2 and 3-pointers, then it would be useful to have a complete dynamic program to generate the exact response to maximize the chance of winning the game. However, since we cannot observe these quantities, our tests of optimality are limited to *comparative statics* and for these tests the myopic assumption is immaterial. Indeed, Proposition 4 can be distilled to the following: as the real value of an asset increases agents will pay a higher nominal price.

# 4 Results

## 4.1 Frequency of 3-point vs. 2-point shot attempts

We first examine the impact of $\alpha$ on the frequency of 3-point shot attempts. We model the probability a team's first shot on a possession is a 3-pointer using a random coefficient linear probability model, which allows coefficients to vary for each team in each season (a "team-year"). We control for each combination of ten players with a unique fixed effect ($\delta$) for each match-up. Our estimating equation is given by:

$$Pr(3\text{PA}_p) = \delta_{Off_p, Def_p} + \beta_{1,t}[\alpha_{p^l} \times 1\{\alpha_{p^l} \leq 1.5\}] + \beta_{2,t}[\alpha_{p^l} \times 1\{\alpha_{p^l} > 1.5\}],$$

where $Off_p$ and $Def_p$ denotes the five-man offensive and defensive line-ups, respectively, on possession $p$ and $\alpha_p$ denotes the value of $\alpha$ faced by the offensive team on possession $p$. This general specification ensures that we are not confounded by lineup effects, but it does chop our data into a very wide (and sometimes very short) unbalanced panel. While the value of $\alpha_p$ is sequentially exogenous to $\text{Points}_p$, it is endogenous to lagged outcomes occurring previously in the same game. This generates a potentially severe dynamic panel bias. In the spirit of the Arellano-Bond panel estimator, we overcome this difficulty by using a lagged value of $\alpha$ calculated on possession $p^l$, the possession of the last substitution event (the last time the FE changed), as a proxy in this and all future econometric specifications.[8]

$\beta_{1,t}$ gives the impact of an increase in $\alpha$ for team $t$ on possessions when the team is leading. Here $\alpha < 1.5$, so as $\alpha$ increases the team should move closer to risk neutrality. $\beta_{2,t}$ gives the impact of an increase in $\alpha$ for team $t$ when they are trailing. Here $\alpha > 1.5$, so as $\alpha$ increases the team

[8] The 'lagged' value, $\alpha_{p^l}$ has a correlation of $\rho = 0.9142$ with $\alpha_p$.

should become more and more risk-loving. Our model without defensive adjustment predicts that an optimizing offensive team should shoot progressively fewer 3's as $\alpha$ increases (as it gains a lead). That is, $\beta_1, \beta_2 < 0$. When we incorporate defensive adjustment, we lose this as a formal prediction. But provided the defensive technology does not fundamentally change with $\alpha$ we should expect both coefficients to have the same sign.

Table 1: Random−coefficient estimates of the impact of $\alpha$ on three−point usage rates.

| Explanatory Variable | Weighted average† coefficient *t−stat* | Mean coefficient *t−stat* | Median coefficient *t−stat*‡ |
|---|---|---|---|
| $\hat{\beta}_1 : \alpha_{p^l} \times (1\{\alpha_{p^l} \leq 1.5\}$ | -0.258 *t=-6.77* | -0.259 *t=-6.30* | -0.262 *t=-4.75* |
| $\hat{\beta}_2 : \alpha_{p^l} \times (1\{\alpha_{p^l} > 1.5\}$ | 0.202 *t=4.62* | 0.216 *t=4.71* | 0.176 *t=3.10* |

Team-years=120, Shots=481,544

† Inverse variance weights used to aggregate coefficients.

‡ Sign test used to construct t−statistics on the median.

Estimating this model for each team-year in our sample produces 120 total estimates of each parameter, which are aggregated in Table 1. Examining the first row, we see that $\beta_1$ is significantly negative ($t = 6.30$, mean), meaning that as the relative real value of 3-pointers increases the leading team shoots fewer of them. As shown in Figure 2, $\alpha$ increasing for the leading team means, all else equal, the game is getting closer. Based on this we would expect $\beta_2$ to be negative as well, meaning the trailing team shoots fewer 3-pointers as they fall further behind. Instead we see that $\beta_2$ is estimated to be significantly positive ($t = 4.71$, mean)—they attempt more 3-pointers as $\alpha$ increases. The response asymmetry is easier to see in Figure 3, which plots semi-parametric estimates (conditioning on lineup fixed effects) aggregated across teams. The magnitudes are meaningful. When a team is in a firmly risk-averse situation they shoot about 20% more 3-pointers as compared to when they should be risk neutral. Provided defensive technology does not depend directly on the score margin the model predicts that this line should be monotonic. Instead we see it is U-shaped, reaching a minimum when the game is tied (precisely when teams should be risk neutral).

## 4.2 The efficiency of 3-point vs. 2-point shot attempts

We delve further into this asymmetry in our analysis of shooting efficiency. Recall that our most robust prediction is given by Proposition 4. Even if players get generally better when they are trailing, our model still implies that 3-point opportunities cannot increase in value as much as 2-point opportunities. That is, the gap in point value between 3 and 2-point attempts must be declining with $\alpha$. Based on what we found in the last subsection, we would not expect this prediction to hold. Since the leading team takes more 3-pointers as they move further into the lead, a downward

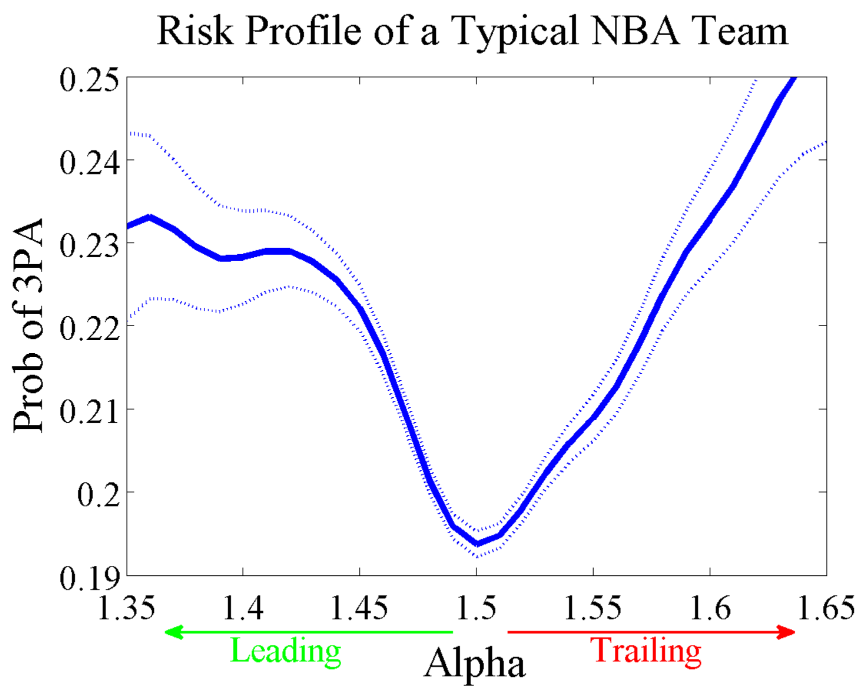


Figure 3: Semi-parametric estimates of 3-point usage rates as a function of $\alpha$ for a typical team. Analysis is conditional on lineup fixed effects and the bandwidth on $\alpha$ is set to .015.

sloping usage function means they would have to accept lower returns to do so. To look at shooting efficiency directly, we use the following random-coefficient linear regression model:

$$
\begin{aligned}
\text{Points}_p = \delta_{Off_p, Def_p} + \beta_{1,t} \cdot 1\{3\text{PA}_p\} + \beta_{2,t}[(\alpha_{p^l} - 1.5) \times 1\{\alpha_{p^l} \leq 1.5\}] + \beta_{3,t}[(\alpha_{p^l} - 1.5) \times 1\{\alpha_{p^l} > 1.5\}] \\
+ \beta_{4,t}[1\{3\text{PA}_p\} \times (\alpha_{p^l} - 1.5) \times 1\{\alpha_{p^l} \leq 1.5\}] + \beta_{5,t}[1\{3\text{PA}_p\} \times (\alpha_{p^l} - 1.5) \times 1\{\alpha_{p^l} > 1.5\}].
\end{aligned}
$$

We again include fixed effects for each unique five-man offensive and defensive line-up to exclude confounding effects from lineup selection. The dependent variable is "total possession efficiency," which is the total number of points scored following the shot attempt but before possession of the ball changes hands (this includes the shot going in, free-throws related to the shot and any points scored after an offensive rebound(s)). In the appendix, we also present coefficient estimates for two related measures: 1) the points scored on the shot for shooters that were not fouled 2) number of points scored on the shot plus any free-throws made related to the shot. The results are not materially different across these measures.

$\beta_1$ can be interpreted as the average nominal point differential between 3- and 2-point shots in a risk-neutral ($\alpha = 1.5$) game state. $\beta_2$ captures the impact of $\alpha$ on 2-pointer efficiency for the leading team ($\alpha \leq 1.5$), while $\beta_3$ captures this effect for the losing team. $\beta_4$ and $\beta_5$ directly test

Proposition 4, these coefficients represent the *differential* effect of $\alpha$ on the efficiency of 3-point attempts relative to 2-point attempts for a winning and losing team respectively.

Estimates of $\beta_{1-5}$ are computed for each team-year. $\beta_1$ is strongly positive—3 pointers have 0.15 higher average point returns in the risk-neutral baseline. Recall that this implies a higher constant and steeper slope for the 3-point usage curve. The estimates for $\beta_2$ and $\beta_3$ are significantly positive. This means that as a team falls further behind, 2-pointers get more efficient. This is strong evidence of the motivational impact of losing.

Table 2: Random−coefficient estimates of the impact of $\alpha_{p^l}$ on nominal returns to 3-point attempts.

| | Total Possession Efficiency | | |
|---|---|---|---|
| Explanatory Variable | Weighted† avg. coeff. | Mean coeff. | Med.‡ coeff. |
| $\hat{\beta}_1 : 1\{3\text{PA}_p\}$ | 0.207 | 0.207 | 0.205 |
| | *t=33.87* | *t=33.27* | *t=10.41* |
| $\hat{\beta}_2 : \alpha^*_{p^l} \times 1\{\alpha_{p^l} \leq 1.5\}$ | 1.07 | 1.26 | 1.13 |
| | *t=5.94* | *t=6.01* | *t=5.29* |
| $\hat{\beta}_3 : \alpha^*_{p^l} \times 1\{\alpha_{p^l} > 1.5\}$ | 0.55 | 0.656 | 0.313 |
| | *t=2.98* | *t=3.27* | *t=2.37* |
| $\hat{\beta}_4 : 1\{3\text{PA}_p\} \times \alpha^*_{p^l}$ | 0.82 | 0.805 | 0.977 |
| $\times 1\{\alpha_{p^l} \leq 1.5\}$ | *t=4.64* | *t=4.23* | *t=3.29* |
| $\hat{\beta}_5 : 1\{3\text{PA}_p\} \times \alpha^*_{p^l}$ | -0.494 | -0.6 | -0.419 |
| $\times 1\{\alpha_{p^l} > 1.5\}$ | *t=-2.70* | *t=-3.17* | *t=-1.83* |

Team-years=120, Shots=481,544.
† Inverse variance weights used to aggregate coefficients
‡ Sign test used to construct t−statistics on the median.
For notational convenience: $\alpha^*_{p^l} = \alpha_{p^l} - 1.5$.

The key test of optimality lies in the estimates of $\beta_4$ and $\beta_5$. Proposition 4 states that optimal response to changing incentives over risk requires that both coefficients are negative. This condition is met for trailing teams, $\beta_5$ is significantly negative ($p < 0.000001$ for the weighted average). Recall from Table 1 that the trailing team also responds to an increase in $\alpha$ by shooting more 3-pointers; this overall pattern of behavior is consistent with the offensive having a greater ability to adjust than the defense. The offense shoots more 3's and the average point value falls as they move down the usage curve, or in other words the offense pays a higher nominal price in forgone 2-pointers for the increase in the real value 3-pointers have in terms of winning the game—trailing offense adheres to the comparative static prediction of its risk-loving utility function. In stark contrast, $\beta_4$ is estimated to be significantly *positive* (and roughly the same absolute value as $\beta_5$). Recall that we found in Table 1 that the leading team tends to shoot fewer 3's when $\alpha$ increases. Here we see that this decrease in usage is accompanied by an increase in efficiency (again consistent with a downward sloping usage curve and limited defensive adjustment). For a leading team, as the game

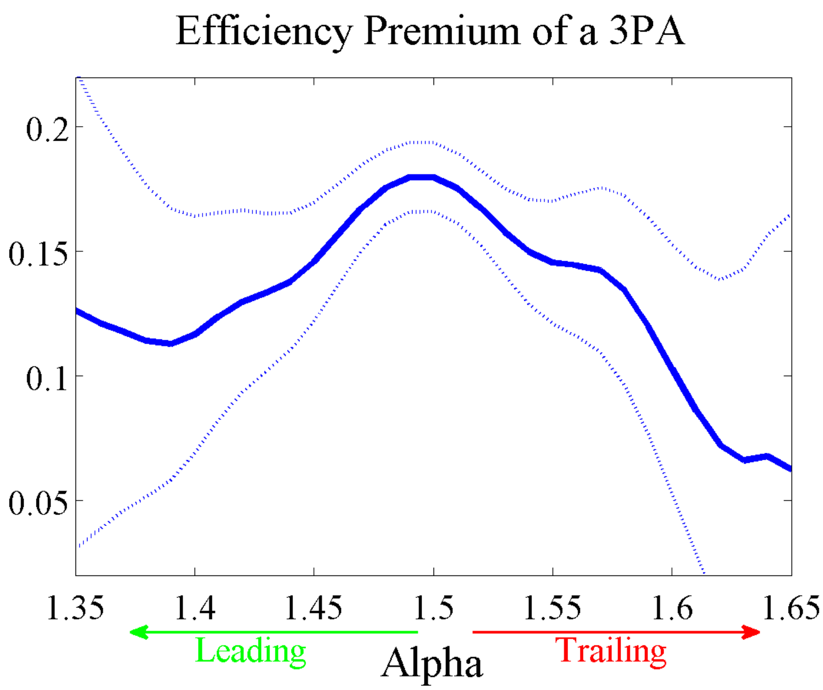


Figure 4: Semi-parametric estimates of 3-point efficiency rates as a function of $\alpha$ for a typical team. Analysis is conditional on lineup fixed effects and the bandwidth on $\alpha$ is set to .015.

gets closer, the team *should* become more risk-neutral, yet the team actually behaves in a more risk averse manner. Leading teams invert the price of risk, paying a greater premium when it is worth less.

The response asymmetry is clearly seen in Figure 4, which plots the efficiency premium of 3-pointers relative to 2-pointers using a semi-parametric estimator that conditions on line-up fixed-effects. The normatively prescribed risk preferences a team should have imply this line ought to be everywhere downward sloping. Instead we see it is only downward sloping the losing domain and is in fact significantly upward sloping in the leading domain. Again the magnitudes are meaningful. The overall estimates indicate nominal 3-point efficiency falls 5-10% when an offense should be substantially risk-averse, this difference separates very good teams from very bad ones in the NBA. Further, the real value drop is even larger given they are also accepting higher per-shot variance with the lower mean efficiency.

## 5 Conclusion

People seem to make choice mistakes by using their "day-to-day" risk preferences in economic contexts that they are ill-suited for. For instance, they buy actuarially unfair extended warranties and are too conservative with their long-term investments. These behaviors often imply absurd levels of risk aversion (Rabin, 2000), but may seem quite reasonable to the person making them. Some of these mistakes are chalked up to inexperience and it is typically difficult to study more experienced agents in the field because the data requirements to study risk attitudes are quite high. One needs an estimate of the ex-ante probabilities and the returns to gambles. Further it is difficult to discern choice mistakes from faithful representations of true preferences. We overcome this challenge by studying a choice setting where the risk preferences agents *should* hold can be pinned down with a single, minimal assumption. These risk preferences are consistent with widely held individual preferences over money when the team is trailing but are inconsistent them when the team is leading. We find shot selection is consistent with optimal play only in trailing domain. When leading, players invert the price of risk, indicating that experience is insufficient to eliminate this type of choice mistake in these agents.

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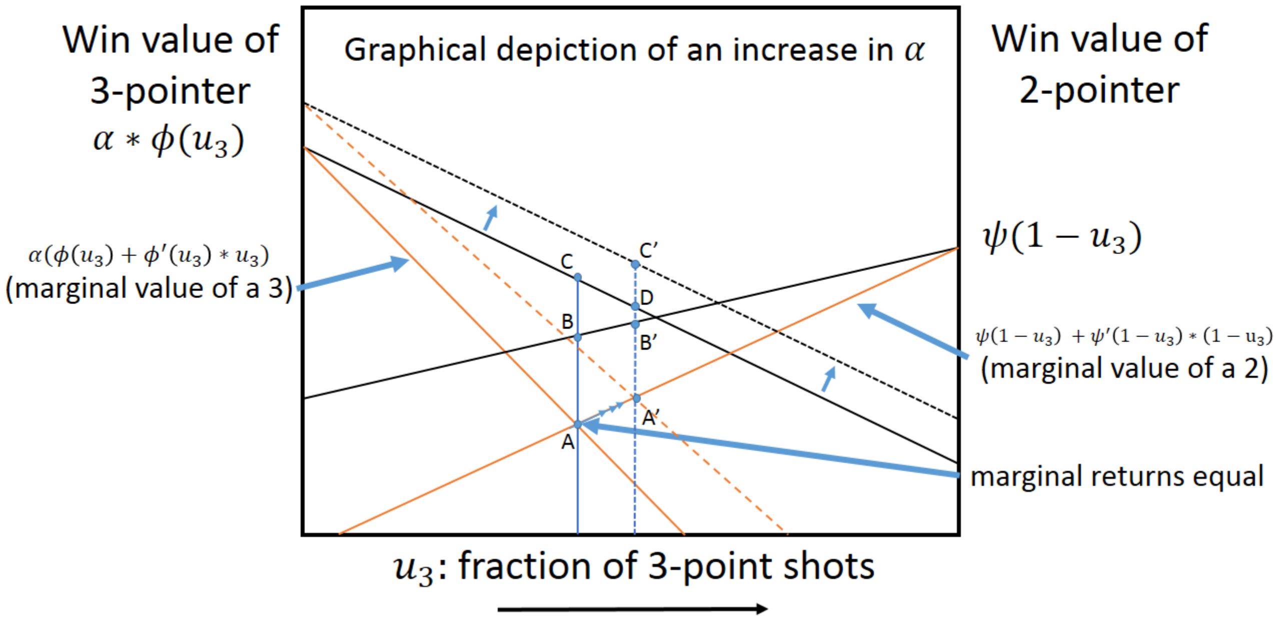
# 6 Appendix

## 6.1 Figures




Appendix Figure 2: Graphical representation of the no-defensive adjustment model. The initial "profit maximizing" condition given by the line intersecting point $A$ and the impact of an increase in $\alpha$ with the new equilbrium given by the line intersecting $A'$.

## 6.2 The model with defensive adjustment

Offense (defense) seeks to maximize (minimize) the offenses increase in win probability in a given possession. This utility function (for the offense) is

$$U = u_3 p_3 WV_3 + u_2 p_2 WV_2$$
$$\frac{U}{WV_2} = \alpha u_3 p_3 + u_2 p_2$$

subject to the constraints that

$$u_2 = 1 - u_3$$
$$d_2 = 1 - d_3$$
$$p_3 = \phi(u_3, d_3)$$
$$p_2 = \psi(u_2, d_2).$$

We assume the following (written in terms of $\phi$ but they all hold for $\psi$ too):

1. Usage curves are downward sloping: $\phi_1 < 0$.
2. Usage curves are such that marginal shots have declining value: $\dfrac{d^2(u_3 \cdot \phi(u_3))}{du_3^2} = 2\phi'(u_3) + u_3\phi''(u_3) < 0$
3. Defensive pressure lowers shooting percentage: $\phi_2, \psi_2 < 0$.
4. Defense has diminishing returns: $\phi_{22}, \psi_{22} > 0$.
5. Using more possessions in a certain way increases (makes more negative) returns to defense against that type of use: $(\phi_2 + u_3^*\phi_{21}) < 0$.

Let starred values denote the equilibrium quantities. Then the defense's first order condition is given by

$$\alpha u_3^* \phi_2(u_3^*, d_3^*) = (1 - u_3^*)\psi_2(1 - u_3^*, 1 - d_3^*), \tag{3}$$

where the subscript denotes a derivative in the corresponding argument. The offense's first order condition is given by

$$\alpha[\phi(u_3^*, d_3^*) + u_3^*\phi_1(u_3^*, d_3^*)] = [\psi(1 - u_3^*, 1 - d_3^*) + (1 - u_3^*)\psi_1(1 - u_3^*, 1 - d_3^*)] \tag{4}$$

where the bracketed quantities represent marginal shot probabilities for 3 and 2 point shots respectively. Both of these must both be greater than 0. Taking total differentiation of (3) and omitting the arguments of $\phi$ and $\psi$ yields

$$u_3^*\phi_2 d\alpha + \alpha u_3^*\phi_{22} dd_3^* + \alpha(\phi_2 + u_3^*\phi_{21})du_3^* = -u_2^*\psi_{22}dd_3^* - (\psi_2 + u_2^*\psi_{21})du_3^*$$

and rearranges to

$$\begin{aligned} u_3^*\phi_2 d\alpha + \left[\alpha(\phi_2 + u_3^*\phi_{21}) + (\psi_2 + u_2^*\psi_{21})\right] du_3^* + [\alpha u_3^*\phi_{22} + u_2^*\psi_{22}]dd_3^* &= 0 \\ b_2 d\alpha + a_{21}du_3^* + a_{22}dd_3^* &\equiv 0, \end{aligned} \tag{5}$$

where the values of $a_{11}, a_{12}$ and $b_1$ are defined implicitly. A similar analysis of equation (4) gives

$$\begin{aligned} [\phi(u_3^*, d_3^*) + u_3^*\phi_1(u_3^*, d_3^*)]d\alpha + [(2\phi_1 + u_3^*\phi_{11}) + (2\psi_1 + u_2^*\psi_{11})]du_3^* & \\ + \left[\alpha\left(\phi_2 + u_3^*\phi_{12}\right) + (\psi_2 + u_2^*\psi_{12})\right] dd_3^* &= 0 \\ b_1 d\alpha + a_{11}du_3^* + a_{12}dd_3^* &\equiv 0. \end{aligned} \tag{6}$$

where the values of $a_{21}, a_{22}$ and $b_2$ defined implicitly. In matrix notation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} du_3^* \\ dd_3^* \end{bmatrix} = - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} d\alpha$$

Then by Cramer's Rule

$$\frac{dd_3^*}{d\alpha} = - \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\overbrace{a_{21}b_1}^{(-)} - \overbrace{a_{11}b_2}^{(+)}}{\underbrace{a_{11}a_{22} - a_{12}a_{21}}_{(-)}} > 0$$

To sign these derivatives not that, $a_{11} < 0$ (marginal shots have diminishing value), $a_{22} > 0$ (diminishing returns to defense), and $a_{12} = a_{21} < 0$ (shooting more 3s raises the effectiveness of defense on 3s). Thus both denominators are negative. Also $b_1 > 0$ (its a marginal shot value) and $b_2 < 0$ ($\phi_2 < 0$). Signing this derivative states that defensive pressure on 3's must increase with $\alpha$.

*Proof of Proposition 3*

$$\frac{du_3^*}{d\alpha} = - \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\overbrace{b_2a_{12}}^{(+)} - \overbrace{b_1a_{22}}^{(+)}}{\underbrace{a_{11}a_{22} - a_{21}a_{12}}_{(-)}} =?.$$

Manipulating the numerator, we have that $\frac{du_3^*}{d\alpha} >$ if and only iff:

$$\frac{a_{12}}{b_1} > \frac{a_22}{b_2}$$

We first note that both sides of this inequality are negative, it is convenient to write:

$$\left|\frac{a_{12}}{b_1}\right| < \left|\frac{a_{22}}{b_2}\right|$$

$a_{12} = \alpha(\phi_2 + u_3^*\phi_{12}) + (\psi_2 + u_2^*\psi_{12})$ is the cross-partial marginal effect of defense. It says "how much more effective does defense become when an offense increases its fraction of 3's. This terms gives the incentive for the defense to adjust into 3's. $b_1$ is the offense's marginal shot value of a 3, as the usage curve gets steeper, this value falls. On the RHS, the numerator is a term, $\alpha u_3^*\phi_{22} + u_2^*\psi_{22}$, that captures the concavity of the defense's response function. The denominator captures the marginal impact of defense. If the above equation holds, the offense will take more 3's when their preference for risk increases. This equation says this is more likely to occur when the defense has a concave adjustment function (they face strong diminishing returns to selective pressure), when the cross partial is low (the extra impact of guarding 3's does not increase much with the offense's 3-point usage) and when the usage curve of a 3-pointer relatively flat (raising the marginal value of a 3-pointer, which raises the denominator on the LHS).

*Proof of Proposition 2*

Other comparative statics are directly implied by our constraints,

$$\begin{aligned}
\frac{du_2^*}{d\alpha} &= -\frac{du_3^*}{d\alpha} = ? \\
\frac{dd_2^*}{d\alpha} &= -\frac{dd_3^*}{d\alpha} < 0 \\
\frac{dp_3^*}{d\alpha} &= \frac{du_3^*}{d\alpha}\phi_1 + \frac{dd_3^*}{d\alpha}\phi_2 \\
&= -\frac{\phi_1(\overbrace{b_1 a_{22}}^{(+)} - \overbrace{b_2 a_{12}}^{(+)}) + \phi_2(\overbrace{a_{11} b_2}^{(+)} - \overbrace{a_{21} b_1}^{(-)})}{\underbrace{a_{11}a_{22} - a_{12}a_{21}}_{(-)}}
\end{aligned}$$

which first does not appear signable, but can be rearranged to

$$= -\frac{\overbrace{\phi_1 b_1 a_{22}}^{(-)} + \overbrace{b_2(\phi_2 a_{11} - \phi_1 a_{12})}^{(-)} - \overbrace{\phi_2 a_{21} b_1}^{(+)}}{\underbrace{a_{11}a_{22} - a_{12}a_{21}}_{(-)}} < 0,$$

where the middle term in the numerator can be signed by noting that $b_2 < 0$ and $(\phi_2 a_{11} - \phi_1 a_{12}) = \phi_1\phi_2(1 + 2u_3^*) > 0$.

$$\frac{dp_2^*}{d\alpha} = \frac{du_2^*}{d\alpha}\psi_1 + \frac{dd_2^*}{d\alpha}\psi_2 < 0,$$

follows by symmetry to the above calculation.

## 6.3 Proofs for the baseline model

*Proof of Proposition 1* The only part of Proposition 1 not shown in the text is that the win value of 3's must increase. We think intuition can be best scene through the lense of a classic economics setup. Consider a monopolist facing demand curve $P(q)$ and an upward slope marginal cost curve $C''(q) < 0$. Imagine a subsidy from the government of so that for each dollar earned, the firm earns $1 + x = \alpha > 1$ dollars. What the proposition states is that if the government offers subsidy $x$, the price cannot fall by more than $x$.

This problem is isomorphic to our shot allocation problem because the downward sloping 2-point usage curve implies an increasing marginal opportunity cost of shooting 3's. As I shoot more 3's, I give up better and better 2-pointers. The first order condition of this problem is:

$$\alpha MR(q) = MC(q)$$

Taking the total derivative, rearranging and multiplying by $\frac{dp}{dq}$ we get:

$$\frac{dp}{d\alpha} = \left(\frac{MR}{MC' - \alpha MR's}\right)\frac{dp}{dq}$$

We are interested in whether $p * \alpha$ is greater than the orginal price, this amounts to whether:

$$\frac{d(p\alpha)}{d\alpha} = \alpha * \frac{dp}{d\alpha} + p > 0$$

Plugging, in our condition becomes, is:

$$p = \left(\frac{\alpha MR}{\alpha MR's - MC'}\right)\frac{dp}{dq}$$
$$p > \frac{(p + p'(q)q)p'(q)}{p''(q)q + 2p'(q)}$$

Cross-multiplying and rearranging we have:

$$qp''(q) < \frac{q(p'(q)^2q - p'(q)p)}{pq}$$
$$-2p'(q) < \frac{q(p'(q)^2q - p'(q)p)}{pq}$$

where the second line follows because $qp''(q) + 2p'(q) < 0$ (marginal revenue is downward sloping). Canceling out and simplying, this equation reduces to:

$$p'(q) > -\frac{p}{q}$$
$$p'(q) * \frac{q}{p} > -1$$
$$\frac{1}{\epsilon} > -1$$

where $\epsilon$ is the elasticity of demand. The last line must hold, otherwise the firm earns negative marginal revenue.

### 6.4 Parametric model of win probability

A game of NBA basketball has 48 minutes of game time, with ties being settled by a 5-minute overtime. Consider two teams, home ($h$) and away ($a$). Let $S_{h,N}$ and $S_{a,N}$ denote the current scores for the home and away team with $N$ offensive possessions (for each team) remaining in the game. Let $P_{h,i}$ and $P_{a,i}$ denote the number of points scored by the home/away team on the $i^{th}$ possession from the end of the game. The home team wins if it has more points at the end of the game, which we can express as:

$$S_{h,0} > S_{a,0} \iff S_{h,N} + \sum_{i=1}^{N} P_{h,i} > S_{a,N} + \sum_{i=1}^{N} P_{a,i} \iff \sum_{i=1}^{N} P_{h,i} - P_{a,i} > S_{a,N} - S_{h,N}.$$

To model how teams generate points, let $\{\mu_h, \sigma_h^2\}$ and $\{\mu_a, \sigma_a^2\}$ represent the mean and variance of points per possession that each team is able to achieve in the match-up. If the number of remaining possessions, $N$, is large, the central limit theorem gives the probability of the home team winning as:

$$\begin{aligned} P(\text{Home Win}) = P(S_{h,0} > S_{a,0}) &= P\left(\sum_{i=1}^{N}(P_{h,i} - P_{a,i}) > S_{a,N} - S_{h,N}\right) \\ &= \Phi\left(\frac{S_{h,N} - S_{a,N} + N(\mu_h - \mu_a)}{\sqrt{N(\sigma_h^2 + \sigma_a^2)}}\right), \end{aligned} \tag{7}$$

where $\Phi$ is the CDF of the standard normal distribution. Examining this expression, we see that an ability advantage ($\mu$ higher than opponent) matters proportional to the number of remaining possessions. Each factor's marginal impact on winning the game is easily obtained by differentiating equation (7). The following expression gives the impact of a point scored for the home team on win probability:

$$\frac{dP(\text{Home Win})}{dS_{h,N}} = \phi\left(\frac{(S_{h,N} - S_{a,N}) + (N(\mu_h - \mu_a))}{\sqrt{N(\sigma_h^2 + \sigma_a^2)}}\right)\frac{1}{\sqrt{N(\sigma_h^2 + \sigma_a^2)}}, \tag{8}$$

where lower-case $\phi$ is the standard normal PDF. To estimate this equation, we first impute the number of remaining possessions using the team-specific paces-of-play in a given match-up and by adding one possession to the team currently holding the ball. Given the standard normal specification, it is natural to estimate equation (7) with Probit regression. The projections give the probability the home team will win at each of state of the game. Figure 1 Panel 1 shows these projections.

## 6.5 Additional Efficiency Metrics

Table 3: Random−coefficient estimates of the impact of $\alpha_{p^l}$ on nominal returns to 3-point attempts.

| | Effective Field Goal % | | | True Shooting % | | | Gross Possession Efficiency | | |
|---|---|---|---|---|---|---|---|---|---|
| Explanatory Variable | Weighted† avg. coeff. | Mean coeff. | Med.‡ coeff. | Weighted† avg. coeff. | Mean coeff. | Med.‡ coeff. | Weighted† avg. coeff. | Mean coeff. | Med.‡ coeff. |
| $\hat{\beta}_1 : 1\{3\text{PA}_p\}$ | 0.207 | 0.207 | 0.205 | 0.127 | 0.127 | 0.13 | 0.142 | 0.142 | 0.142 |
| | *t=33.87* | *t=33.27* | *t=10.41* | *t=21.49* | *t=21.13* | *t=9.31* | *t=24.00* | *t=23.51* | *t=9.68* |
| $\hat{\beta}_2 : \alpha^*_{p^l} \times 1\{\alpha_{p^l} \leq 1.5\}$ | 1.07 | 1.26 | 1.13 | 0.806 | 0.975 | 0.82 | 0.927 | 1.12 | 1.16 |
| | *t=5.94* | *t=6.01* | *t=5.29* | *t=4.77* | *t=4.94* | *t=4.56* | *t=5.49* | *t=5.66* | *t=4.75* |
| $\hat{\beta}_3 : \alpha^*_{p^l} \times 1\{\alpha_{p^l} > 1.5\}$ | 0.55 | 0.656 | 0.313 | 1.04 | 1.17 | 0.923 | 1.5 | 1.55 | 1.78 |
| | *t=2.98* | *t=3.27* | *t=2.37* | *t=6.03* | *t=6.29* | *t=4.38* | *t=8.68* | *t=8.33* | *t=6.02* |
| $\hat{\beta}_4 : 1\{3\text{PA}_p\} \times \alpha^*_{p^l}$ | 0.82 | 0.805 | 0.977 | 1.09 | 1.1 | 1.33 | 1.11 | 1.11 | 0.839 |
| $\times 1\{\alpha_{p^l} \leq 1.5\}$ | *t=4.64* | *t=4.23* | *t=3.29* | *t=6.46* | *t=6.02* | *t=4.38* | *t=6.58* | *t=6.10* | *t=4.02* |
| $\hat{\beta}_5 : 1\{3\text{PA}_p\} \times \alpha^*_{p^l}$ | -0.494 | -0.6 | -0.419 | -0.936 | -1.05 | -0.887 | -0.888 | -0.949 | -0.899 |
| $\times 1\{\alpha_{p^l} > 1.5\}$ | *t=-2.70* | *t=-3.17* | *t=-1.83* | *t=-5.41* | *t=-5.86* | *t=-3.65* | *t=-5.14* | *t=-5.30* | *t=-3.83* |

Team-years=120, Shots=481,544, † Inverse variance weights used to aggregate coefficients
‡ Sign test used to construct t−statistics on the median. * We suppress the −1.5.